

## A COMMON FIXED POINT THEOREM FOR FOUR SELF MAPPINGS IN FUZZY METRIC SPACE

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### ABSTRACT

The purpose of this paper is to establish a common fixed point theorems in fuzzy metric space using the concept of weak compatible mappings for four self-mapping and generalizing the result of Sharma et.al[8]. We also cite an example in support of our result.

**Keywords:** Common fixed point, Fuzzy metric space, R-weakly commuting maps, compatible mappings, weak compatible mappings.

**AMS (2000) Subject Classification:** 54H25, 47H10.

### INTRODUCTION

The concept of fuzzy set was introduced by Zadeh (1965) [12] as a new way to represent vagueness in everyday life. A large number of renowned Mathematicians worked with fuzzy sets in different branches of Mathematics. One such is the Fuzzy Metric Space. In this paper we use the concept of fuzzy metric space introduced by Kramosil and Michalek [4] and modified by George and Veeramani [1] with the help of t-norm. Grabiec [5] obtained the fuzzy version of Banach contraction principle, which is a milestone in developing fixed point theory in fuzzy metric space. In the sequel, Vasuki [7] introduced the concept of R-weakly commuting in fuzzy metric space and proved the common fixed point theorem. The concept of compatibility was generalized by Jungck [3]. The concept of compatibility in fuzzy metric space was proposed by Mishra *et. al.*, [6]. In 1996 Jungck again generalized the notion of compatible mapping by introducing weak mapping [2]. In 2005, Singh and Jain [10] introduced the concept of semi-compatible mappings and proved the common fixed point theorem for four self-mapping in fuzzy metric space. That was generalization of Vasuki[7].

In 2012, Sharma *et. al.*, [8] also generalized the result for three commuting mappings instead of two mapping, given by Vasuki [7].

In this paper we generalize the result for four weak compatible mappings instead of three mapping, given by Sharma *et. al.*, [8]. Our result is also a generalization of Singh and Jain [10].

### 1. PRELIMINARIES

**Definition 2.1.** [12] Let  $X$  be any set. A fuzzy set  $A$  in  $X$  is a function with domain in  $X$  and values in  $[0, 1]$ .

**Definition 2.2.** [6] A binary operation  $*$  on  $[0, 1]$  is called a continuous t-norm if it satisfies the following conditions:

- (i)  $*$  is associative and commutative,
  - (ii)  $*$  is continuous,
  - (iii)  $a * 1 = a$ , for all  $a \in [0, 1]$ ,
  - (iv)  $a * b \leq c * d$ , whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0, 1]$ .
- Examples of t-norms are

- \*  $a * b = \min \{a, b\}$
- \* (minimum t-norm),  $a * b = \min \{a, b\}$
- ab (product t-norm).

**Definition 2.3.** [1] The 3-tuple  $(X, M, *)$  is called a fuzzy metric space if  $X$  is an arbitrary set,  $*$  is a continuous t-norm and  $M$  is a fuzzy set on  $X^2 \times (0, \infty)$  satisfying the following conditions:

- (FM-1)  $M(x, y, t) > 0$ ,
- (FM-2)  $M(x, y, t) = 1$  if and only if  $x = y$ ,
- (FM-3)  $M(x, y, t) = M(y, x, t)$ ,
- (FM-4)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ ,
- (FM-5)  $M(x, y, \cdot) : (0, \infty) \rightarrow [0, 1]$  is continuous, for all  $x, y, z \in X$  and  $t, s > 0$ .

Let  $(X, d)$  be a metric space and let  $a * b = ab$  or  $a * b = \min \{a, b\}$  for all  $a, b \in [0, 1]$ .

$$M(x, y, t) = \frac{t}{t + d(x, y)} \text{ for all } x, y \in X \text{ and } t > 0.$$

Then  $(X, M, *)$  is a fuzzy metric space, and this fuzzy metric  $M$  induced by  $d$  is called the standard fuzzy metric [1].

**Definition 2.4.** [6] A sequence  $\{x_n\}$  in a fuzzy metric space  $(X, M, *)$  is said to be convergent to a point  $x \in X$ , if  $\lim_{n \rightarrow \infty} M(x_n, x, t) = 1$  for all  $t > 0$ .

Further, the sequence  $\{x_n\}$  is said to be Cauchy if

$$\lim_{n \rightarrow \infty} M(x_n, x_{n+p}, t) = 1 \text{ for all } t > 0 \text{ and } p > 0.$$

The space  $(X, M, *)$  is said to be complete if every Cauchy sequence in  $X$  is convergent in  $X$ .

**Lemma 2.5.** [5] Let  $(X, M, *)$  be a fuzzy metric space. Then  $M$  is non-decreasing for all  $x, y \in X$ .

**Lemma 2.6.** [6] Let  $(X, M, *)$  be a fuzzy metric space. Then  $M$  is a continuous function on  $X^2(0, \infty)$ .

Throughout this paper  $(X, M, *)$  will denote the fuzzy metric space with the following condition:

$$(FM-6) \quad \lim_{n \rightarrow \infty} M(x, y, t) = 1 \text{ for all } x, y \in X \text{ and } t > 0.$$

**Definition 2.7.** [7] Two mappings  $f$  and  $g$  of a fuzzy metric space  $(X, M, *)$  into itself

$M(fgx, gfx, t) \geq M(fx, gx, t)$  for each  $x$  in  $X$ .  
are said to be weakly commuting if

**Definition 2.8.** [8] Two mappings  $f$  and  $g$  of a fuzzy metric space  $(X, M, *)$  into itself are said to be  $R$ -weakly commuting if

$$M(fgx, gfx, t) \geq M\left(fx, gx, \frac{t}{R}\right) \text{ for all } x \text{ in } X$$

**Remark:**  $R$ -weakly commutativity implies weak commutativity only when  $R \leq 1$ .

**Definition 2.9.** [9] Let  $f$  and  $g$  be self-mappings on a fuzzy metric space  $(X, M, *)$ . The pair  $(f, g)$  is said to be compatible if

$$\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t) = 1 \text{ for all } t > 0,$$

whenever  $\{x_n\}$  is a sequence in  $X$  such that  $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z$  for some  $z \in X$ .

**Definition 2.10.** [11] Let  $f$  and  $g$  be self-mappings on a fuzzy metric space  $(X, M, *)$ . Then the mappings are said to be weakly compatible if they commute at their coincidence point,

that is,  $fx = gx$  implies  $fgx = gfx$ .

It is known that a pair of  $(f, g)$  compatible maps is weakly compatible but converse is not true in general.

**Definition 2.11.** [10] A pair  $(A, B)$  of self maps of a fuzzy metric space  $(X, M, *)$  is said to be semi-compatible if  $\lim_{n \rightarrow \infty} M(ABx_n, Ax_n) = 1$ , whenever  $\{x_n\}$  is a sequence in  $X$  such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = x.$$

It follows that if  $(A, B)$  is semi-compatible and  $Ax = Bx$  then  $ABx = BAx$  that means every semi-compatible pair of self-maps is weak compatible but the converse is not true in general.

In 2012, Sharma et al. [8] proved the following result:

**Theorem 2.12.** Let  $f, g$  and  $h$  be three self-mappings on a fuzzy metric space  $(X, \mathcal{M}, *)$  satisfying following (2.11.1) & (2.11.2) conditions. Suppose that  $h$  is continuous and pairs  $(f, h)$  &  $(g, h)$  are R-weakly commuting on  $X$ . Then  $f, g$  and  $h$  have a unique common fixed point in  $X$ .

$$(2.12.1) \quad f(X) \cap g(X) \subset h(X).$$

$$(2.12.2) \quad M(fx, gy, t) \geq r\{M(hx, hy, t)\} \text{ for all } x, y \text{ in } X.$$

Where  $r : [0, 1] \rightarrow [0, 1]$  is a continuous function such that  $r(t) > t$  for each  $0 < t < 1$ .

### 3. MAIN RESULT

Our result generalizes the results of Sharma et al. [8] as we are using the concept of weak compatibility, which are lighter conditions than R-weakly commuting, compatibility and semi compatibility. We are proving the result for four self-mappings in a fuzzy metric space such that only one weak compatible pair is sufficient.

**Theorem 3.1:** Let  $(X, \mathcal{M}, *)$  be a complete fuzzy metric space and let  $P, Q, A, B$  be mappings from  $X$  into itself such that the following conditions are satisfied

$$(3.1.1) \quad P(X) \cap Q(X) \subset AB(X),$$

$$(3.1.2) \quad (P, AB) \text{ or } (Q, AB) \text{ is weak-compatible,}$$

$$(3.1.3) \quad AB = BAB = BP.$$

$$(3.1.4) \quad \text{For every } x, y \text{ in } X \text{ and } t > 0,$$

$$M(Px, Qy, t) \geq rM(ABx, AB y, t)$$

Where  $r : [0, 1] \rightarrow [0, 1]$  is a continuous function such that  $r(t) > t$

for each  $0 < t < 1$ .

Then  $P, Q, A$  and  $B$  have a unique common fixed point in  $X$ .

**Proof:** Let  $x$  be an arbitrary point in  $X$ .

According to (3.1.1), there exist some points  $x_1, x_2 \in X$  such that

$$Px_0 = ABx_1 = y_0 \text{ and } Qx_1 = ABx_2 = y_1.$$

We can construct sequences  $\{x_{2n}\}$  and  $\{x_{2n+1}\}$  in  $X$  such that

$$y_{2n} = Px_{2n} = ABx_{2n+1} \text{ and } y_{2n+1} = Qx_{2n+1} = ABx_{2n+2} \text{ for } n=0,1,2,\dots$$

Now, we first show that  $\{y_n\}$  is a Cauchy sequence in  $X$ .

Using condition (3.1.4)

$$M(y_{2n+1}, y_{2n}, t) = M(Px_{2n}, Qx_{2n+1}, t)$$

$$\begin{aligned} M(y_{2n+1}, y_{2n}, t) &\geq rM(ABx_{2n}, ABx_{2n+1}, t), \\ &\geq rM(y_{2n-1}, y_{2n}, t) \end{aligned}$$

$$M(y_{2n+1}, y_{2n}, t) \geq M(y_{2n-1}, y_{2n}, t) \dots (i)$$

$$\text{Similarly } M(y_{2n+2}, y_{2n+1}, t) \geq M(y_{2n}, y_{2n+1}, t) \dots (ii)$$

From (i) and (ii)

Therefore  $\{M(y_{n+1}, y_n, t)\}$  is an increasing sequence of positive real numbers in  $[0, 1]$  and tends to limit  $l \leq 1$ . Now we prove that  $l = 1$ .

Let us suppose that  $l < 1$  then,  $M(y_{n+1}, y_n, t) \geq r M(y_n, y_{n-1}, t)$ .

On taking  $n \rightarrow \infty$  we get

$$\lim_{n \rightarrow \infty} M(y_{n+1}, y_n, t) \geq r \lim_{n \rightarrow \infty} M(y_n, y_{n-1}, t).$$

Now for any positive integer  $p$ ,

$$M(y_n, y_{n+p}, t) \geq M(y_n, y_{n+1}, t/p) * M(y_{n+1}, y_{n+2}, t/p) * \dots * M(y_{n+p-1}, y_{n+p}, t/p).$$

Hence, the subsequences  $\{Px_{2n}\}, \{Qx_{2n+1}\}, \{ABx_{2n}\}$  and  $\{ABx_{2n+2}\}$  also converge to  $z$ .

$$\lim_{n \rightarrow \infty} Px_{2n} = z = \lim_{n \rightarrow \infty} ABx_{2n+2} = z.$$

**Case (1)** When  $(P, AB)$  is weak compatible mappings,

$$\lim_{n \rightarrow \infty} P x_{2n} = z \text{ and } \lim_{n \rightarrow \infty} A B x_{2n+1} = z,$$

Then there exist  $u \in X$  such that  $A B u =$

Step (I) Putting  $u$  and  $y = x_{2n+1}$  in condition [3.1.4]

$$M(Pu, Qx_{2n+1}, t) \geq rM(ABu, ABx_{2n+1}, t)$$

$$M(Pu, Qx_{2n+1}, t) \geq rM(z, ABx_{2n+1}, t)$$

Let  $n$  and using above result we get

$$M(Pu, z, t) \geq rM(z, z, t) > M(z, z, t)$$

$$M(Pu, z, t) \rightarrow 1 \text{ as } n \rightarrow \infty$$

$$Pu = z$$

$$Pu = ABu = z.$$

This result so that 'u' is coincident point of X such that  $Pu = ABu =$

$$Pu = ABu = z \text{ and } (\Phi, AB) \text{ weak compatible mappings}$$

$$PABu = ABPu.$$

$$ABu = z \rightarrow PABu = Pz.$$

$$Pu = z \rightarrow ABPu = ABz$$

$$Pz = ABz. \quad \dots \text{(iii)}$$

Putting  $z$  and  $y = x_{2n+1}$  in [3.1.4]

$$M(Pz, Qx_{2n+1}, t) \geq rM(ABz, ABx_{2n+1}, t)$$

$$M(Pz, Qx_{2n+1}, t) \geq rM(Pz, ABx_{2n+1}, t)$$

and using above result we get  $\square \square \square \square$  Letting  $n$

$$M(Pz, z, t) \geq rM(Pz, z, t).$$

$\Rightarrow M(Pz, z, t) > M(Pz, z, t)$  which is contradiction

Hence,  $Pz = z$ .

Using (iii) we get  $ABz = Pz = z$ . ....(iv)

Now again putting  $x = x_{2n}$  and  $y = z$  in [3.1.4]

$$M(Px_{2n}, Qz, t) \geq rM(ABx_{2n}, ABz, t)$$

$$M(Px_{2n}, Qz, t) \geq rM(ABx_{2n}, z, t)$$

and using above result we get  $\square\square\square\square$  Letting  $n$

$$M(z, Qz, t) \geq rM(z, z, t)$$

$$> M(z, z, t)$$

$$Qz = z. \quad \dots(v)$$

$$Pz = Qz = z = ABz \text{ since (iii), (iv) and (v)}$$

**Case (2).** When  $(Q, AB)$  is weak compatible mappings,

$\lim_{n \rightarrow \infty} Qx_{2n+1} = z$  and  $\lim_{n \rightarrow \infty} ABx_{2n} = z$ , Then there exist  $v \in X$  such that  $ABv = z$ .

Putting  $x = x_{2n}$  and  $y = v$  in condition [3.1.4]

$$M(Px_{2n}, Qv, t) \geq rM(ABx_{2n}, ABv, t)$$

$$M(Px_{2n}, Qv, t) \geq rM(ABx_{2n}, z, t)$$

Letting  $n$  and using above result we get

$$M(z, Qv, t) \geq rM(z, z, t)$$

$$M(z, Qv, t) \geq r(1)$$

$$Qv = z$$

$$Qv = ABv = z.$$

'v' is a coincident point of X such that  $Qv = ABv = z$

Now,  $Qv = ABv = z$  and  $(Q, AB)$  weak compatible mappings.

$$QABv = ABQv$$

$$ABv = z \rightarrow QABv = Qz.$$

$$Qv = z \rightarrow ABQv = ABz$$

$$Qz = ABz \dots \dots \dots (vi)$$

Now again putting  $x = x_{2n}$  and  $y = z$  in [3.1.4]

$$M(Px_{2n}, Qz, t) \geq rM(ABx_{2n}, ABz, t)$$

$$M(Px_{2n}, Qz, t) \geq rM(ABx_{2n}, Qz, t), \text{ since (vi)}$$

Letting  $n \rightarrow \infty$  and using above result we get

$$M(z, Qz, t) \geq rM(z, Qz, t)$$

$$M(z, Qz, t) > M(z, Qz, t), \text{ which is contradiction}$$

Hence  $Qz = z$ .

$$Qz = ABz = z.$$

Putting  $x = z$  and  $y = x_{2n+1}$  in [3.1.4]

$$M(Pz, Qx_{2n+1}, t) \geq rM(ABz, ABx_{2n+1}, t)$$

$$M(Pz, Qx_{2n+1}, t) \geq rM(z, ABx_{2n+1}, t).$$

Letting  $n \rightarrow \infty$  and using above result we get

$$M(Pz, z, t) \geq rM(z, z, t)$$

$$\Rightarrow M(Pz, z, t) > r(1), \text{ which is contradiction}$$

Hence,  $Pz = z$ .  $Pz = Qz = z = ABz$

Now, according to case (1) and case (2) we get  $Pz = Qz = z = ABz \dots \dots (vii)$

Putting  $x = z$  and  $y = x_{2n+1}$  in  $M(PBz, Qx_{2n+1}, t) \geq rM(AB(Bz), ABx_{2n+1}, t)$

$$M(PBz, Qx_{2n+1}, t) \geq rM(AB(Bz), ABx_{2n+1}, t) \quad [3.1.4]$$

$$M(BPz, Qx_{2n+1}, t) \geq rM(BABz, ABx_{2n+1}, t), \text{ since (3.1.3)}$$



$$M(Bz, Qx_{2n+1}, t) \geq rM(Bz, ABx_{2n+1}, t), \text{ Since (vii)}$$

Letting  $n \rightarrow \infty$  and using above result we get

$$M(Bz, z, t) \geq rM(Bz, z, t).$$

$$Bz = z$$

$$Bz = z \text{ and } ABz = z$$

$$Az = z$$

Thus,  $Pz = Qz = Az = Bz = z$ .

$z$  is a common fixed point of mapping  $P, Q, A$  and  $B$ .

Uniqueness:

Let 'w' be another common fixed point of  $P, Q, A$  and  $B$ .

$$\text{Then } Pz = Qz = Az = Bz = z \text{ and } Pw = Qw = Aw = Bw = w.$$

Putting  $x = z$  and  $y = w$  in [3.1.4]

$$M(Pz, Qw, t) \geq rM(ABz, ABw, t)$$

$$M(z, w, t) \geq rM(Az, Aw, t)$$

$$M(z, w, t) \geq rM(z, w, t)$$

$$z = w.$$

Therefore 'z' is unique common fixed point of  $P, Q, A$  and  $B$  mappings of  $X$ .

**Remark 3.2.** If we take  $B = I$  (identity mapping) in theorem 3.1 then the condition (3.1.3) is satisfied trivially and we get the following result.

**Corollary 3.3:** Let  $P, Q$  and  $A$  be self-mappings of complete fuzzy

metric space  $(X, M, *)$ . Suppose that the following conditions

are satisfied:

$$(3.1.1) \quad P(X) \cap Q(X) \subset A(X),$$

(3.1.2) Pair  $(P, A)$  or  $(Q, A)$  is

weakcompatible, (3.1.3)

For every  $x, y$  in  $X$  and  $t > 0$ ,

$$M(Px, Qy, t) \geq rM(Ax, Ay, t).$$

Where  $r: [0,1] \rightarrow [0,1]$  is a continuous function such that  $r(t) > t$  for each  $0 < t < 1$ .

Then  $P, Q$  and  $A$  have a unique common fixed point in  $X$ .

**Example 3.4:** Let  $X = [0, 3]$  and  $(X, d)$  be a metric space where metric  $d(x, y) = |x - y|$ .

Define  $a * b = a \cdot b$   $a, b \in [0,1]$  and  $M$  fuzzy set on  $X^2 \times (0, \infty)$  such that

$$M(x, y, t) = \frac{|x - y|}{t + |x - y|}$$

We take three functions  $f, g, h: [0,3] \rightarrow [0,3]$  define as follow:

$$f(x) = \begin{cases} x & x \in [0, 1) \\ 3 & x \in [1, 3] \end{cases}, g(x) = 3 \text{ for all } x \in [0,3] \quad \text{and} \quad h(x) = \begin{cases} 3 - x & x \in [0, 1) \\ 3 & x \in [1, 3] \end{cases}$$

Then  $f(X) = [0, 1) \cup \{3\}$ ,  $g(X) = \{3\}$  and  $h(X) = (2, 3]$ .

Therefore,  $f(X) \cap g(X) \subseteq h(X)$

When  $x \in [0, 1]$   $f(x) = x$  and  $h(x) = 3 - x$

$fh(x) = f(3 - x) = 3$  and  $hf(x) = h(x) = 3 - x$ .

$$M(f(x), hx, t) = \frac{|x - (3 - x)|}{t + |x - (3 - x)|} = \frac{|2x - 3|}{t + |2x - 3|}$$

$$M(fh(x), hf(x), t) = \frac{|3 - (3 - x)|}{t + |3 - (3 - x)|} = \frac{|x|}{t + |x|}$$

When putting  $x = \frac{1}{2} \in [0, 1]$

$$M\left(f\left(\frac{1}{2}\right), h\left(\frac{1}{2}\right), t\right) = \frac{2}{t + 2}$$

$$M\left(f\left(\frac{1}{2}\right), h\left(\frac{1}{2}\right), \frac{t}{2}\right) = \frac{2}{(t/2) + 2} = \frac{4}{t + 4}$$

And

$$M\left(fh\left(\frac{1}{2}\right), hf\left(\frac{1}{2}\right), t\right) = \frac{1/2}{t + 1/2} = \frac{1}{2t + 1}$$

Hence, we get  $M\left(f\left(\frac{1}{2}\right), h\left(\frac{1}{2}\right), t/2\right) > M(fh(1/2), hf(1/2), t)$ .

So that  $M(fh(x), hf(x), t/2) > M(fx, hx, t)$ , is not satisfied for all  $x \in [0, 1]$ .

Hence,  $(f, h)$  is not R-Weakly commuting mapping but  $(f, h)$  is weak compatible mapping. Because  $f(x) = h(x)$  when  $x \in [1, 3]$

Then there exist a coincident point in  $[0, 3]$  such that  $fh(x) = hf(x)$  when  $x \in [1, 3]$

$(f, h)$  is commuting at coincident points.

Similarly  $(g, h)$  is also weak compatible  $g(x) = h(x)$  when  $x \in [1, 3]$

$$g(x) = h(x) \Rightarrow gh(x) = hg(x) \text{ when } x \in [1, 3]$$

$$g(3) = h(3) = 3 \Rightarrow gh(3) = hg(3)$$

Then exist a common unique fixed point 3 in X such that  $f(3) = g(3) = h(3) = 3$ .

Remark:

Example (3.3) shows that  $(f, h)$  is not R-Weakly commuting mapping but there exist a unique common due to the weak compatibility. Every R-Weakly commuting mapping is weak compatible but converse is not true.

### CONCLUSION

This paper is generalization of the result of Sharma et.al. [8] in the sense of replace R-Weakly commuting to weakly compatible (owc) to prove a theorem on common fixed point theorems for four self mappings in complete fuzzy metric space.

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